TIME DOMAIN SIMULATION OF UNIFORM AND NONUNIFORM MULTICONDUCTOR LOSSY LINES BY THE METHOD OF CHARACTERISTICS

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Abstract: A numerical technique to compute the time domain response of multiconductor lossy uniform and nonuniform lines terminated in general nonlinear elements is presented. The technique is based on the generalized method of characteristics and is used to study signal delay, distortion and crosstalk in interconnections in integrated circuits and chip carriers.

Introduction

A number of techniques have been formulated in recent years to compute the time domain response of multiconductor uniform and nonuniform lines [1–7]. The work reported on lossy lines, however, has been limited to the special cases of homogeneous medium [4], analysis based on frequency domain solutions [5], or other special cases (e.g., [6]) where coupling between non-adjacent lines is neglected. The method of characteristics, which has been applied to lossless and lossy distortionless lines [7], transforms the initial pair of equations into a pair of ordinary differential equations. Each of these two ordinary differential equations holds true along a family of *characteristic curves*, often abbreviated to characteristics. For the lossless and lossy distortionless line these ordinary differential equations can be integrated directly [7]. For the lossy line the equations can be solved numerically. In the paper the method is expanded and applied to study time domain response of single and multiple coupled lossy uniform and nonuniform lines including interconnections in integrated circuits and chip carriers (Fig. 1).

Theory

The voltages and currents on a nonuniform multiconductor transmission line are given by

$$\frac{\partial \mathbf{e}}{\partial x} + \mathbf{L}(x)\frac{\partial \mathbf{i}}{\partial t} + \mathbf{R}(x)\,\mathbf{i} = \mathbf{0}, \tag{1}$$

$$\frac{\partial \mathbf{i}}{\partial x} + \mathbf{C}(x)\frac{\partial \mathbf{e}}{\partial t} + \mathbf{G}(x)\,\mathbf{e} = \mathbf{0}, \tag{2}$$

together with the boundary conditions at $x = 0$ and $x = d$. Here, $\mathbf{e} = \mathbf{e}(x,t)$ and $\mathbf{i} = \mathbf{i}(x,t)$ are n-dimensional voltage and current vectors, respectively, and $\mathbf{R}(x)$, $\mathbf{L}(x)$, $\mathbf{G}(x)$, $\mathbf{C}(x)$ are real, symmetric $n \times n$ matrices.

Let the terminations, or boundary conditions, be given by

$$\mathbf{i}(0,\, t) = \mathbf{f}(\mathbf{v}_g(t),\, \mathbf{e}(0,\, t),\, t), \tag{3}$$

$$\mathbf{i}(d,\, t) = \mathbf{g}(\mathbf{v}_s(t),\, \mathbf{e}(d,\, t),\, t), \tag{4}$$

where $\mathbf{v}_g(t)$ and $\mathbf{v}_s(t)$ are the vectors of generator voltages at the input ($x = 0$) and output ($x = d$) ends of the line and f and g are well behaved vector functions (f and g can be extended to describe arbitrary terminations).

In order to find the solution of the system of equations (1)–(2), it is first transformed into a system having diagonal **L** and **C** matrices. This can be done as follows:
Let $\mathbf{e}_T$ and $\mathbf{i}_T$ be transformed voltage and current vectors related to **e** and **i** by

$$\mathbf{e} = \mathbf{E}(x)\,\mathbf{e}_T\,, \quad \mathbf{i} = \mathbf{H}(x)\,\mathbf{i}_T\,, \tag{5}$$

where $\mathbf{E}(x)$ and $\mathbf{H}(x)$ are $n \times n$ matrices.
Substituting (5) into (1)–(2) gives

$$\frac{\partial \mathbf{e}_T}{\partial x} + \mathbf{L}_T\frac{\partial \mathbf{i}_T}{\partial t} + \mathbf{R}_T\,\mathbf{i}_T + \mathbf{E}^{-1}\frac{d\mathbf{E}}{dx}\mathbf{e}_T = \mathbf{0}, \tag{6}$$

$$\frac{\partial \mathbf{i}_T}{\partial x} + \mathbf{C}_T\frac{\partial \mathbf{e}_T}{\partial t} + \mathbf{G}_T\,\mathbf{e}_T + \mathbf{H}^{-1}\frac{d\mathbf{H}}{dx}\mathbf{i}_T = \mathbf{0}, \tag{7}$$

where

$$\mathbf{L}_T = \mathbf{L}_T(x) = \mathbf{E}^{-1}(x)\,\mathbf{L}(x)\,\mathbf{H}(x), \tag{8}$$

$$\mathbf{R}_T = \mathbf{R}_T(x) = \mathbf{E}^{-1}(x)\,\mathbf{R}(x)\,\mathbf{H}(x), \tag{9}$$

$$\mathbf{C}_T = \mathbf{C}_T(x) = \mathbf{H}^{-1}(x)\,\mathbf{C}(x)\,\mathbf{E}(x), \tag{10}$$

$$\mathbf{G}_T = \mathbf{G}_T(x) = \mathbf{H}^{-1}(x)\,\mathbf{G}(x)\,\mathbf{E}(x). \tag{11}$$

The matrices $\mathbf{E}(x)$ and $\mathbf{H}(x)$ can be determined from the requirement that $\mathbf{L}_T(x)$ and $\mathbf{C}_T(x)$ have to be diagonal. Using (8) and (10):

$$(\mathbf{LC})\,\mathbf{E} = \mathbf{E}\,(\mathbf{L}_T\mathbf{C}_T), \tag{12}$$

$$(\mathbf{CL})\,\mathbf{H} = \mathbf{H}\,(\mathbf{C}_T\mathbf{L}_T). \tag{13}$$

It is seen from (6) and (7) that **E** and **H** are the eigenvector matrices of **LC** and **CL** matrices, respectively. Since **L** and **C** are symmetrical matrices, transposing (13) shows that **H** is related to **E** by $\mathbf{H} = (\mathbf{E}^T)^{-1}$, where $\mathbf{E}^T$ denotes the transpose of **E**.

The boundary conditions given by equations (3) and (4) transform to

$$\mathbf{i}_T(0, t) = \mathbf{H}^{-1}(0)\, f(\mathbf{v}_g(t), \mathbf{E}(0)\, \mathbf{e}_T(0, t), t\,), \quad (14)$$

$$\mathbf{i}_T(d, t) = \mathbf{H}^{-1}(d)\, g(\mathbf{v}_s(t), \mathbf{E}(d)\, \mathbf{e}_T(d, t), t\,). \quad (15)$$

The Generalized Method of Characteristics

The system of partial differential equations (6) and (7) is reduced to a system of ordinary differential equations which are valid along a family of curves in the (x,t) plane. This system of ordinary differential equations is then solved numerically.

$$\text{If} \qquad x = x_1(t),\ x = x_2(t),\ \ldots,\ x = x_n(t) \quad (16)$$

are the equations of n curves in the (x,t) plane, then dx_k/dt (k = 1, 2, ..., n) are the slopes of the tangents to these curves and

$$d\mathbf{e}_T = \mathbf{dx}\frac{\partial \mathbf{e}_T}{\partial x} + dt\frac{\partial \mathbf{e}_T}{\partial t}, \quad d\mathbf{i}_T = \mathbf{dx}\frac{\partial \mathbf{i}_T}{\partial x} + dt\frac{\partial \mathbf{i}_T}{\partial t}, \quad (17)$$

where e_{Tk}, i_{Tk} (k = 1, 2, ..., n) are the elements of $\mathbf{e}_T$, $\mathbf{i}_T$ and $\mathbf{dx}=\mathrm{diag}(dx_1, \ldots, dx_n)$. By substituting equation (17) into (6) and (7) it can be shown that there are two choices of curves (16) that will transform them into ordinary differential equations. The two choices are given by

$$\frac{\mathbf{dx}}{dt} = \pm\,[\mathbf{L}_T(x)\,\mathbf{C}_T(x)]^{-1/2}, \quad (18)$$

where $\frac{\mathbf{dx}}{dt} = \mathrm{diag}\,(\frac{dx_1}{dt}, \frac{dx_2}{dt}, \ldots, \frac{dx_n}{dt})$. Along these families of characteristic curves the system (6) and (7) can be reduced leading to the system of equations given by

$$\mathbf{I}\,dt\; - (\mathbf{C}_T\mathbf{L}_T)^{1/2}\,\mathbf{dx} = \mathbf{0}, \quad (19)$$

$$d\mathbf{e}_T + \mathbf{Z}_0\, d\mathbf{i}_T + \mathbf{dx}\Big[\mathbf{Z}_0(\mathbf{G}_T\,\mathbf{e}_T + \mathbf{H}^{-1}\frac{d\mathbf{H}}{dx}\mathbf{i}_T\,) + \mathbf{R}_T\,\mathbf{i}_T + \mathbf{E}^{-1}\frac{d\mathbf{E}}{dx}\mathbf{e}_T\Big] = \mathbf{0}, \quad (20)$$

$$\mathbf{I}\,dt\; + (\mathbf{C}_T\mathbf{L}_T)^{1/2}\,\mathbf{dx} = \mathbf{0}, \quad (21)$$

$$d\mathbf{e}_T - \mathbf{Z}_0\, d\mathbf{i}_T + \mathbf{dx}\Big[-\mathbf{Z}_0(\mathbf{G}_T\,\mathbf{e}_T + \mathbf{H}^{-1}\frac{d\mathbf{H}}{dx}\mathbf{i}_T\,) + \mathbf{R}_T\,\mathbf{i}_T + \mathbf{E}^{-1}\frac{d\mathbf{E}}{dx}\mathbf{e}_T\Big] = \mathbf{0}, \quad (22)$$

where **I** is the identity matrix.

Numerical Solution

The above system of equations can be solved numerically by approximating the differentials by finite differences. The resulting system of equations is solved for the voltage and current vectors at successive time points. The procedure is outlined below (Fig. 2).

I Choose the number of points where the voltages are to be calculated (m+1). This specifies Δx in Fig. 2 as $\Delta x = d/m$.

II Choose Δt as

$$\Delta t = \min_{\substack{i \in \{1\ldots n\} \\ \text{over all } x}} [\Delta x \sqrt{L_T^{(i)}(x)\, C_T^{(i)}(x)}\,]. \quad (23)$$

III At points along the line

1. $\mathbf{x}(R_i)$ and $\mathbf{x}(S_i)$ are calculated from the finite difference approximations of (19) and (21).

2. $\mathbf{e}_T(R_i)$, $\mathbf{i}_T(R_i)$, $\mathbf{e}_T(S_i)$, $\mathbf{i}_T(S_i)$ are calculated from known $\mathbf{e}_T$ and $\mathbf{i}_T$ at points C, D, E at time t using linear interpolation.

3. $\mathbf{e}_T(P)$ and $\mathbf{i}_T(P)$ are obtained from the finite difference approximation of equations (20) and (22).

4. $\mathbf{e}(P)$ and $\mathbf{i}(P)$ can then be obtained from

$$\mathbf{e}(P) = \mathbf{E}(D)\,\mathbf{e}_T(P)\,, \quad \mathbf{i}(P) = \mathbf{H}(D)\,\mathbf{i}_T(P). \quad (24)$$

IV At $x = 0$

1. $\mathbf{x}(U_i)$ is calculated from the finite difference approximation of (21).

2. Calculate $\mathbf{e}_T(U_i)$, $\mathbf{i}_T(U_i)$ from $\mathbf{e}_T(A)$, $\mathbf{e}_T(B)$, $\mathbf{i}_T(A)$ and $\mathbf{i}_T(B)$ using linear interpolation.

3. Solve the finite difference approximation of (20) together with (14) for $\mathbf{e}_T(U_i)$ and $\mathbf{i}_T(U_i)$. This can be done by some iterative method.

4. Transform back to get $\mathbf{e}(M)$, $\mathbf{i}(M)$:

$$\mathbf{e}(M) = \mathbf{E}(A)\,\mathbf{e}_T(M)\,, \quad \mathbf{i}(M) = \mathbf{H}(A)\,\mathbf{i}_T(M). \quad (25)$$

V At x = d the calculations proceed in a similar manner as for x=0.

Steps III–V are then repeated for each time point.

Results and Concluding Remarks

The step response of typical uniform and nonuniform structures have been computed by utilizing the technique outlined above. For lossless uniformly coupled lines the step response is found to be in agreement with those reported in [1] and [2]. All the calculations were performed on a desktop computer. The capacitance, inductance, resistance and conductance matrices can be computed by utilizing various available numerical techniques such as those based on finite difference, boundary element and spectral domain methods [1,9,10]. The diagonal resistance matrix elements in these examples are taken as constant corresponding to the low frequency quasi-static solution.

Figure 3 shows the step response of an asymmetric nonuniformly coupled microstrip structure. The circuit was analyzed by using the above procedure with 100 divisions per line. The computed step response of a uniformly coupled three line structure with nonlinear terminations is shown in Fig. 4, together with the structure geometry and the schematic. The diode is defined by $i = I_s(e^{v/V_T} - 1)$, where $I_s = 1$ nA and $V_T = 25$ mV. The circuit was analyzed by using 100 divisions per line. In order to demonstrate that the properties of nonuniformly coupled interconnections, such as those encountered in certain chip carriers, can be computed by using the technique, Fig. 5 shows the step response of a symmetric nonuniform coupled three line structure. The near and far end crosstalk has a behavior similar to that of uniformly coupled lines [1, 2].

In conclusion, a numerical technique based on the generalized method of characteristics has been developed to compute the time domain response of general lossy multiconductor coupled uniform and nonuniform lines. The technique should be helpful in the analysis and design of interconnections in high speed digital and analog hybrid and monolithic circuits, chip carriers and packages.

References

[1] V. K. Tripathi and R. J. Bucolo, *IEEE Trans. Electron Devices*, pp. 650–658, March 1987.

[2] V. K. Tripathi and J. B. Rettig, *IEEE Trans. MTT*, pp. 1513–1518, Dec. 1985.

[3] Y. E. Yang, J. A. Kong and Q. Gu, *IEEE Trans. MTT*, pp. 1120–1130, Nov. 1985.

[4] F. Y. Chang, *IEEE Trans. MTT,* pp. 2028–2038, Dec. 1989.

[5] A. R. Djordjevic et. al., *Proc. IEEE*, pp. 743–764, June 1987.

[6] D. S. Gao et. al., *IEEE Trans.Circuits and Sys.* pp. 1–9, Jan. 1990.

[7] F. H. Branin, Jr., *Proc IEEE*, pp. 2012–2013, Nov. 1967.

[8] M. Lister, "The numerical solution of hyperbolic partial differential equations by the method of characteristics", in Mathematical Methods for Digital Computers, A.Ralston and H.S. Wilf, New York : Wiley, 1960, chap. 15.

[9] H. Lee and V. K. Tripathi, *IEEE MTT-S Intl. Microwave Symp.*, pp. 5710–573, June 1983.

[10] A. R. Djordjevic, et. al., "Matrix Parameters of Multiconductor Transmission Lines", Artec House, 1989.

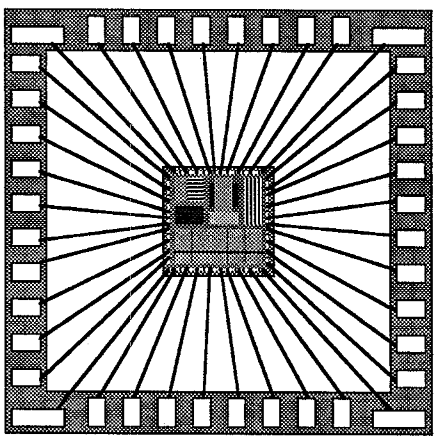

Figure 1: Generic example of multiconductor lines

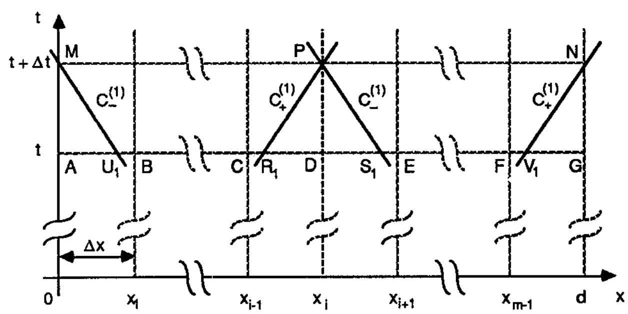

Figure 2: Computation of voltages and currents on the line

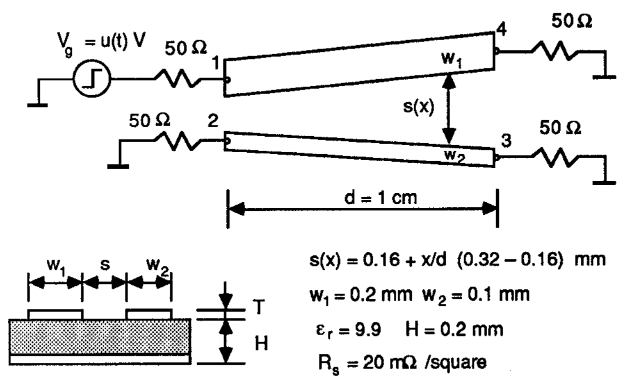

Figure 3(a): Crossectional view and schematic of asymmetric coupled nonuniform lines

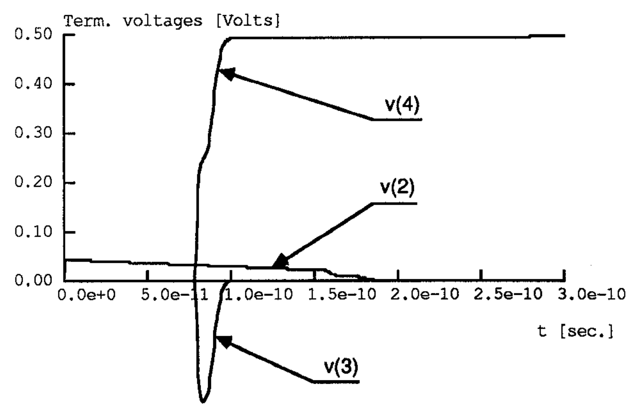

Figure 3(b) : Step response of the coupled line four port of Fig. 3(a).

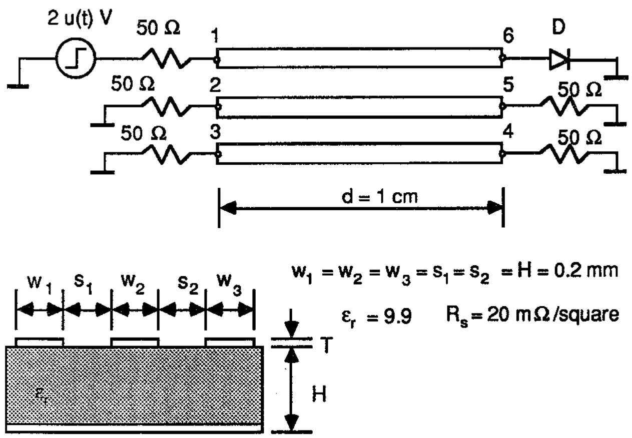

Figure 4(a): Crossectional view and schematic of symmetrical uniformly coupled microstrip line six port terminated in nonlinear element.

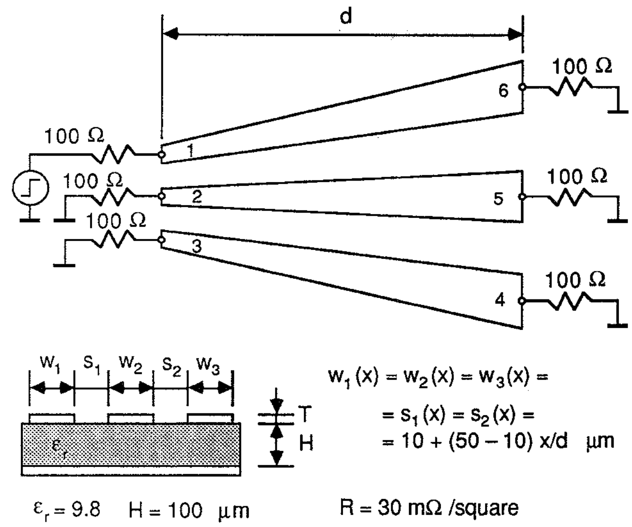

Figure 5(a): Schematic of three nonuniformly coupled microstrips.

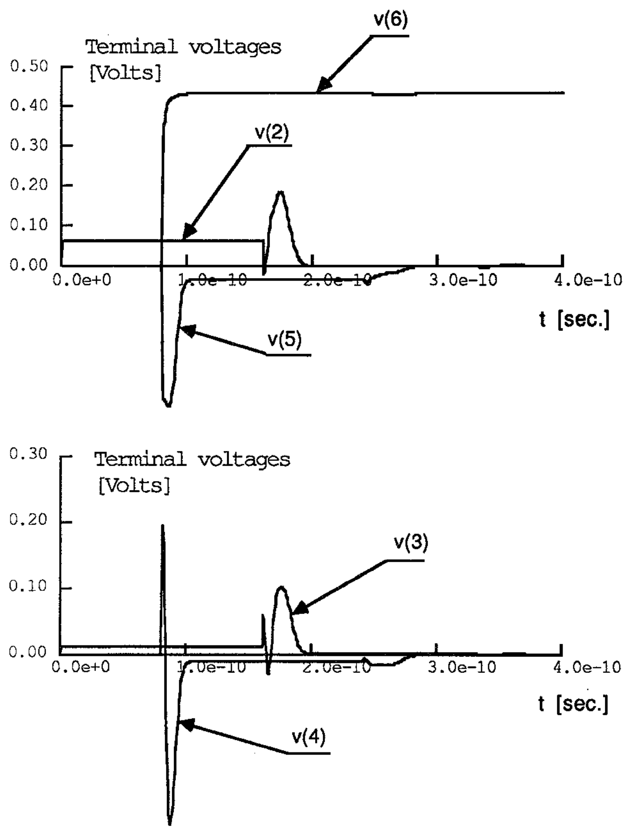

Figure 4(b): Step response of the three line uniform structure.

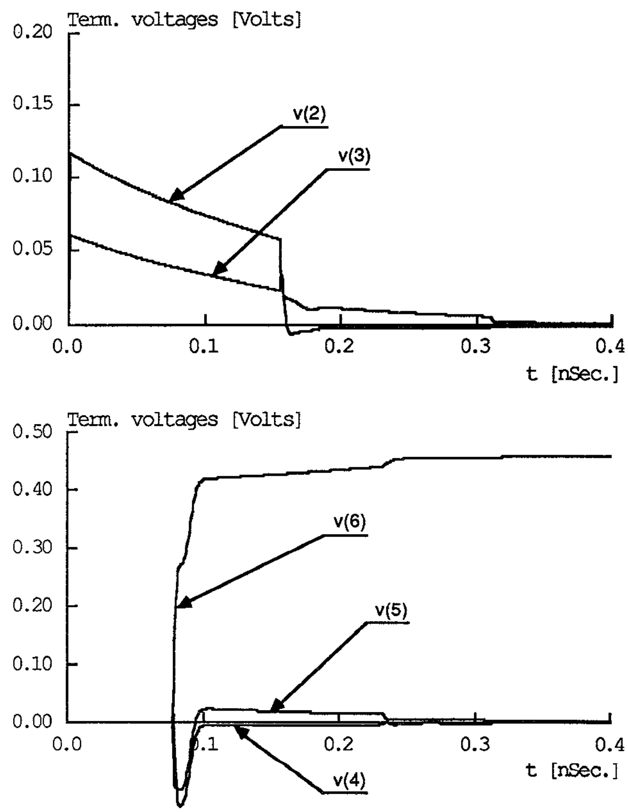

Figure 5(b): Step response of the nonuniform three-line structure.